SYSTEM THEORETIC APPROACH TO NMR IMAGING

by

KOMARAVOLU PHANI RAJ KUMAR

DEPARTMENT OF ELECTRICAL ENGINEERING

INDIAN INSTITUTE OF TECHNOLOGY, KANPUR

MARCH 1986

SYSTEM THEORETIC APPROACH TO NMR IMAGING

A Thesis Submitted
In Partial Fulfilment of the Requirements
for the Degree of

MASTER OF TECHNOLOGY

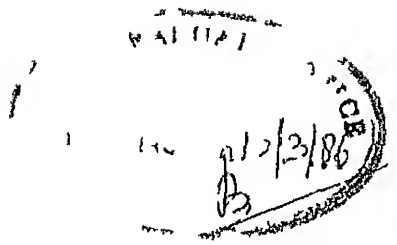
by

KOMARAVOLU PHANI RAJ KUMAR

to the

DEPARTMENT OF ELECTRICAL ENGINEERING
INDIAN INSTITUTE OF TECHNOLOGY, KANPUR

MARCH 1986



CERTIFICATE

This is to certify that the thesis titled 'System Theoretic Approach to NMR Imaging' submitted by Mr Komaravolu Phani Raj Kumar, Roll No 8410422, in partial fulfilment of the requirements for the award of M Tech degree, has been carried out under our supervision and has not been submitted elsewhere for a degree

(K R Srivathsan)
Assistant Professor
Department of Electrical Engg
Indian Institute of Technology
Kanpur

(K R. Sarma)
Professor
Department of Electrical Engg
Indian Institute of Technology
Kanpur

March, 1986

ACKNOWLEDGEMENTS

I express my deep sense of gratitude to Dr K R Sarma for the guidance and encouragement I sincerely thank Dr K R Srivathsan for many useful discussions I had with him

I acknowledge with pleasure the help, in the form of literature and suggestions, provided by Dr Bernhard Blumich, Max-Planck Institut for Polymerforschung, Mainz

I thank Dr P Raghunathan for sailing me through the initial phase of NMR My thanks are due to Dr P T Narasimhan's NMR group for allowing me to have a free access to their literature

Finally, I acknowledge the excellent typing work of Mr C M Abraham

Komaravolu Phani Raj Kumar

ABSTRACT

The work presented in this thesis relates to the signal processing for NMR imaging The problem of NMR imaging is viewed as a system identification problem The Bloch equation governing the NMR imaging is treated as state variable representation of the system to be identified The RF signal excitation is treated as the input and the gradient signals are viewed as system parameters Thus the NMR imaging system is a linear time varying system, and the identification problem amounts to finding the initial state given the response (FID signal) over an interval of time The existing modalities of NMR imaging are explained in this point of view

NMR chirp imaging, based on stochastic imaging is proposed This scheme of imaging exhibits important merits over other pulse sequences One is simplicity in generating gradient functions In NMR imaging the quest for high spatial resolution is of fundamental concern in clinical applications The proposed chirp imaging permits slow switching of field gradients, admitting stronger gradients for improved resolution In addition this scheme may permit, using charge coupled device hardware thereby reducing the hardware complexity

Computer simulation is carried out for obtaining the FID signal for stochastic excitation and chirp imaging The appropriate signal processing techniques are simulated and the processed images are found to be in close agreement with the assumed models

CONTENTS

CHAPTER 1

INTRODUCTION

Nuclear Magnetic Resonance (NMR) is an indispensable tool for material characterization in many disciplines Recently it has been applied as a method of internal imaging, and in this role NMR enjoys advantages like non-invasive, penetrating bony structure without any attenuation, non hazardous nature, high resolution capability, potential for specific imaging

In this chapter we discuss the principles of NMR and its applications in imaging context The hardware associated with NMR imaging system, historical development and system theory approach to NMR imaging is presented The NMR imaging problem can be viewed as a system identification problem,which is well studied in communication and control area and this view point is elaborated in this thesis

1 1 PRINCIPLES OF NMR IMAGING [13]

Nuclear Magnetic Resonance is based on the fact that the atomic nuclei of most elcments possess a 'spin' which in turn is associated with a small nuclear 'magnetic moment' The nucleus, e g , the hydrogen atom or proton, can therefore be imagined as behaving like a tiny bar magnet or compass needle (a magnetic dipole) In the absence of an external magnetic

field the directions of these dipoles have a random orientation (Fig 1 1(a)),but with the application of an external static magnetic field the nuclei begin to precess describing a conical path around the magnetic field direction with a frequency proportional to field strength (Fig 1 1(b)) This frequency is called nuclear magnetic resonance frequency or Larmor frequency Energy considerations favour orientation of the nuclear dipoles in a direction parallel to the applied field direction and the majority of dipoles precess such that their points tend more towards this direction The sum of these elementary dipoles results in a macroscopic magnetization ($\vec{M}$) Applying a radio frequency (RF) pulse brings all the nuclear spins into phase All the precessing dipoles in phase, produces an alternating magnetic field capable of inducing into a pick-up coil an alternating current of resonating frequency which could be measured When the RF pulse is removed, detection of the nuclear signal induced into the receiver coil by free precession is then possible This induced signal is known as a Free Induction Decay (FID) signal

The resonance phenomenon can be visualized from quantum mechanical description of NMR (Fig 1 2) When proton nuclei are placed in a magnetic field, the nuclei will be in two energy states, namely + μH and - μH states [7], where μ and H

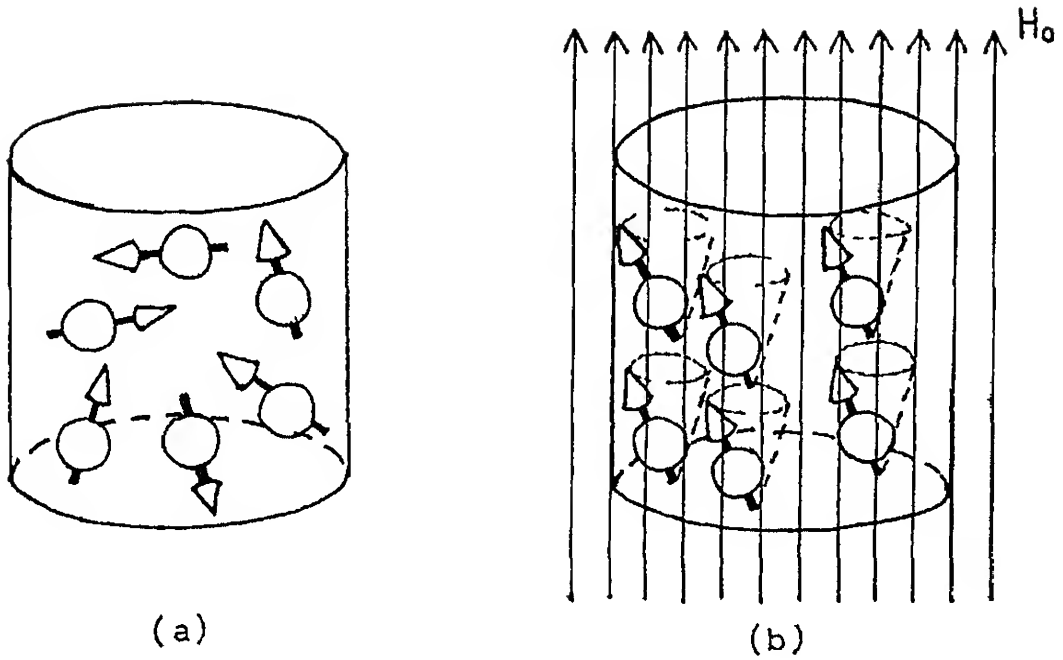


Fig 1 1 Behaviour of nuclear spins (a) without magnetic field (b) with strong magnetic field

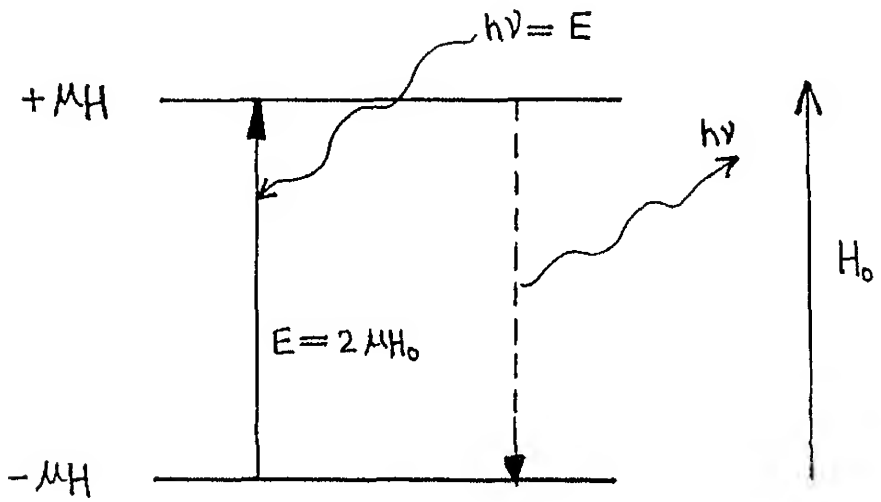


Fig 1 2 Quantum mechanical description of NMR spin excitation

are the nuclear magnetic moment and applied magnetic field, respectively For those nuclei in -μH energy state, irradiation of electromagnetic radiation of energy E equivalent to 2 μH will raise the proton energy state upto +μH This energy is usually supplied by an RF magnetic field

If an auxiliary magnetic field is applied by means of gradient coils so that, for example, the field of the magnet increases linearly from left to right then the protons at the extreme left of the body will have the lowest nuclear resonance frequency, with the frequency increasing linearly to the right depending on the gradient field, thus the spatial coordinates are mapped to frequency scale The amplitude of the NMR signal at a particular frequency, hence, is proportional to the number of nuclear spins (e g , protons) on resonance at the corresponding spatial coordinate

To obtain information about the structure of more complex three-dimensional form (like human head) having within it an uneven distribution of proton concentrations, magnetic field gradients need to be applied in all directions so that frequency values may be assigned to coordinates throughout the object

The relaxation mechanisms associated with excited nuclear spins are transverse relaxation (T_2) and longitudinal relaxation (T_1) T_2 is faster than T_1 so that T_2 is always smaller than T_1

The characterization of tissue matter is in terms of water content Water is the medium in which most of the body chemistry or metabolism takes place, hence it plays an essential role in all biological systems [18] The reason why water has so far played a minor role in medical diagnosis is simply the fact that until the advent of NMR imaging, there really was no way of studying it in vivo Although NMR imaging will doubtless find applications in the study of plants and fluid transport, emphasis is placed here on the study of biological tissue in man Biological tissue contains on average around 75/ water (Fig 1 3), which can readily be detected by NMR methods Quite a large fraction of water is contained within the cell cytoplasm but there is also a substantial amount of extra cellular water in addition to the bulk fluids contained within the body Different healthy body tissues contain different amounts of water and variations in water content can be used to some extent to differentiate the various human tissues although the variations between tissues are rather small

It is also found experimentally that there are differences in the spin relaxation times of various tissues and there is a good correlation between the spin-lattice relaxation time T_1 and water content among the various animal tissues, the longer T_1's being observed in tissues with higher water content Damadian [8] has shown that T_1 is longer in malignant tissue

Fig 1.3 Water content, relaxation time in various human tissues

by a factor of about 1 5 to 2 0 Such pathological tissue is also found to contain more water It would thus seem from the tissue characteristics that an imaging method based on water content and/or T_1 would easily differentiate between the various tissue types yielding cross-sectional images showing anatomical detail as reflected in the distribution of water throughout the body The spin-spin relaxation time T_2, which also varies with tissue type, might be similarly used, given suitable modification of the imaging method Imaging capabilities of these two important parameters, T_1 and T_2, together with the spin densities make NMR imaging a unique, versatile, and powerful technique in medical imaging

1 2 BASIC NMR IMAGING COMPONENTS [11]

A typical NMP imaging system designed for human imaging is shown in Fig 1 4 [7] The basic comoonents of an NMR imaging are as follows

1) A magnet, which provides a strong (range - 500 G to 15 KG), uniform, static magnetic field (H_o)
2) An RF transmitter, which delivers radio frequency (range - 2 to 70 MHz for proton imaging) magnetic fields to the sample ($H_1(t)$)
3) A gradient system, which produces time varying magnetic fields (range - 0 01 to 1 G/cm) of controlled spatial nonuniformity ($G_x(t)$, $G_y(t)$, $G_z(t)$)

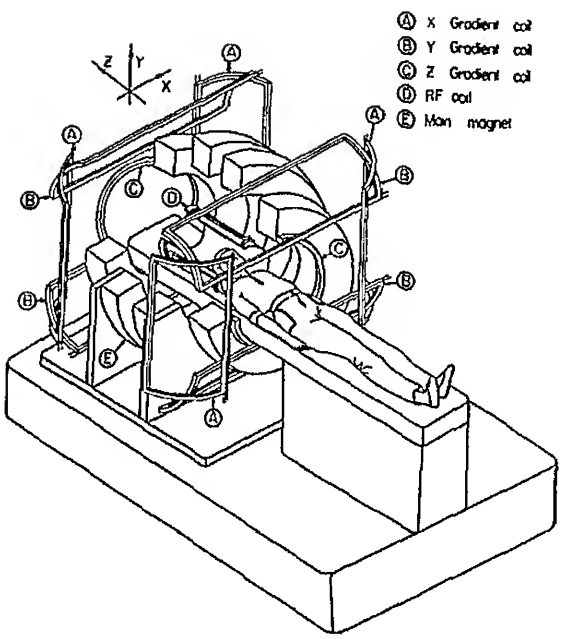


Fig 1.4 Sketch of NMR tomograph [7]

4) A detection system, which yields the output signal (y(t))

5) An imager system, which reconstructs and displays the images

Imaging system shown in Fig 1.4 uses a single RF coil which is used for both transmission of RF signal (longest RF pulse duration is on the order of 2 ms) and reception of FID signal (like a Pulsed Radar) The stochastic imaging discussed in the next section assumes two coils one for transmission and other for reception, since transmission and reception are done simultaneously

Resolution in a conventional NMR imaging system depends on the field homogeneity and available strength of field gradients For example, with a magnet of the static magnetic field of H_o = 5 KG having homogeneity of ΔH = 4 ppm (0 02 G) with field gradient of $G_{x,y \text{ or } z}$ = 0 4 G/cm, the expected resolution limit is approximately 0 5 mm The physical arrangement of components and instrumentation is an important factor in NMR tomography

1 3 HISTORICAL BACKGROUND [18]

At the beginning of 1970's both Lauterbur [16] and Damadian [8] showed that NMR spectroscopic techniques can be applied to imaging of the human body and that the techniques have diagnostic uses similar to those of X-ray computerized

tomography (CT) In 1978, Andrew et al demonstrated the very-high-resolution capability of NMR by obtaining a fine detail image of the submillimeter septum of a small orange More recently, human head images shown by Moore and Holland et al have definitely demonstrated the great potential of NMR imaging in medical imaging Already the subject has acquired a variety of names of which more common are NMR imaging, spin mapping, Zeugmatography, NMR tomography

1 4 SYSTEM THEORY APPROACH

It is well known that a linear time invariant (LTI) system is completely characterized by its impulse response System identification problem is equivalent to the impulse response determination Various methods used in impulse response determination of LTI systems can be broadly classified as sinusoidal wave testing, pulse testing and cross correlation method These are the familiar techniques already in usage in NMR In correlation method an input signal with impulsive autocorrelation function is applied to the system and the response is crosscorrelated with shifted versions of input signal to obtain impulse response point by point

In case of linear time varying (LTV) systems, response of the system for an arbitrary input cannot be given in a closed form as in the case of LTI systems But in general when infinite series solution is obtained under conditions which permit

the system to be viewed as under perturbation, the solution resembles that of a LTI system In NMR imaging equations governing the system behaviour (Bloch equation) correspond to that of LTV system under the conditions, the input RF excitation is small [12] the response resembles the familiar convolution integral and hence emenable to cross correlation approach The output of cross correlation operation gives the information regarding the spatial spin distribution [2] Details of this approach are given in Chapter 2

1 5 SCOPE OF THIS THESIS

Chapter 2 develops the mathematical model of NMR imaging problem, NMR imaging is treated in the system theoretic point of view as LTV system, described by state representation Solution of the state equations is discussed Chapter 3 covers the cross correlation techniques to process the FID signal to obtain the spatial spin distribution Chapter 4 discusses the computer simulation of NMR imaging system model and the signal processing schemes FID signal corresponding to specified gradients and RF excitation are generated by computer using the model Associated cross correlation scheme is applied to obtain the assumed spin distribution Chapter 5 concludes the thesis with suggestions for future work

CHAPTER 2

MATHEMATICAL MODEL

2.1 BLOCH EQUATION

The Bloch equation [1], gives an accurate but phenomenological description of the time dependence of the 'nuclear magnetization' M(t) in the presence of an applied magnetic field H(t) The nuclear magnetization M(t) is the source of the 'NMR Signal' from which the image is ultimately constructed The NMR sample is treated as a black box with H(t) as the input signal, or stimulus, and M(t) as the output signal or response The black box is characterized by $M_o(x,y,z)$, T_1 and T_2, and its behaviour by the Bloch equation

$$\frac{d\bar{M}}{dt} = \gamma \bar{M} \times \bar{H} - \left(\frac{M_x u_x + M_y u_y}{T_2}\right) - \left(\frac{M_z - M_o}{T_1}\right) u_z \qquad (2.1)$$

The above equation can be conveniently represented in matrix notation as

$$\begin{bmatrix} \dot{M}_x \\ \dot{M}_y \\ \dot{M}_z \end{bmatrix} = \begin{bmatrix} -\frac{1}{T_2} & \gamma h & 0 \\ -\gamma h & -\frac{1}{T_2} & \gamma H_1(t) \\ 0 & -\gamma H_1(t) & 0 \end{bmatrix} \begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix}$$

List of Symbols

$\bar{\int}$	Three dimension Integral
Im	Imaginary part of
Re	Real part of
H_o	Strength of the static magnetic field
γ	Gyromagnetic ratio For protons its value is 4.26×10^7 Hz Tesla^{-1}
$h(t,x,y,z)$	Gradient magnetic field $(G_x(t)x + G_y(t)y + G_z(t)z)\, u_z$ Its effect in u_x, u_y is neglected because u_z term dominates
$H(t,x,y,z)$	Total magnetic field experienced at t,x,y,z (represented as $\bar{H}$ or $H(t)$) It has components in u_x, u_y and u_z directions
$H_1(t)$	Input RF signal Its effect is only u_x direction
$M(t,x,y,z)$	Sample magnetization at t,x,y,z (represented as $\bar{M}$ or $M(t)$)
M_x, M_y, M_z	Components of $\bar{M}$
$M = M_x + jM_y$	(Complex) transverse magnetization
$M_o(x,y,z)$	Equilibrium magnetization or spin density
$\omega_o = -\gamma H_o$	Larmour frequency corresponding to the static field
ω	Irradiation frequency of the RF field
u_x, u_y, u_z	Unit vectors in x,y,z directions respectively
$G_x(t), G_y(t), G_z(t)$	Gradient magnetic fields applied along x,y,z axes respectively

The nuclear magnetization $\vec{M}(t)$, induced in the sample by the magnetic field $H(t)$, is the local sum of the magnetic fields of the protons The magnetization is a bulk property of the sample rather than that of individual protons

2 2 EXCITATION METHODOLOGIES

Two kinds of excitations are seen in NMR imaging First, short-pulse excitation in which first a RF pulse, sufficient to tilt magnetization into transverse plane, is applied At the end of RF pulse, field gradients are applied during which FID signal is recorded Second, both excitation and gradients are applied simultaneously throughout the experiment and FID signal is recorded

Short Pulse excitation method can be conveniently explained through the Bloch equation discussed earlier The system dynamics after the application of excitation pulse is given by

$$\begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix} = \begin{bmatrix} -\frac{1}{T_2} & \gamma h & 0 \\ -\gamma h & -\frac{1}{T_2} & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix}$$

and initial conditions change from

$$\begin{bmatrix} 0 \\ 0 \\ M_o(x,y,z) \end{bmatrix}$$

to

$$\begin{bmatrix} M_{x_o}(x,y,z) \\ M_{y_o}(x,y,z) \\ 0 \end{bmatrix}$$

where $M_o(x,y,z) = \sqrt{M_{x_o}(x,y,z)^2 + M_{y_o}(x,y,z)^2}$ Solution for M can be given as

$$M = e^{-t/T_2} e^{-j\gamma \int_o^t h(t')dt'} M_{xy_o}(x,y,z)$$

where $M_{xy_o}(x,y,z) = M_{x_o}(x,y,z) = -jM_{y_o}(x,y,z)$

It is interesting to note that similar solution is arrived at in stochastic NMR imaging

2 3 PERTURBATION ANALYSIS [12]

In this method both input excitation and gradients are considered at same time

Equation (2 1) can be written as

$$\frac{dM_x}{dt} = -\frac{M_x}{T_2} + M_y \gamma h \qquad (2\ 2)$$

$$\frac{dM_y}{dt} = -M_x \gamma h - \frac{M_y}{T_2} + M_z \gamma H_1(t) \qquad (2\ 3)$$

$$\frac{dM_z}{dt} = -M_y \gamma H_1(t) - \left(\frac{M_z - M_o}{T_1}\right) \qquad (2\ 4)$$

Let $M = M_x + jM_y$

The equations (2 2), (2 3) and (2 4) may be written, neglecting spin-lattice relaxation (i e , $t << T_1$) in the form

$$\frac{dM}{dt} + (\frac{1}{T_2} + j\gamma h)M = j\gamma H_1(t)\, M_z \qquad (2\ 5)$$

$$\frac{dM_z}{dt} \approx -\gamma H_1(t)\ \mathrm{Im}\ M \qquad (2\ 6)$$

Due to linear time varying characteristics of a driven NMR system it is usual to discard the simple methods of Fourier analysis applicable to other spectroscopic forms The particular case considered here is the response of the NMR system to an arbitrary shaped pulse of duration less than T_2 The approach to be employed is a combination of perturbation theory and linear system analysis [19]

Let us recall that M is a complex quantity, M_x+jM_y, and M_z real These can be expanded as

$$M = M^{(o)} + M^{(1)} + M^{(2)} + M^{(3)} + \qquad (2\ 7)$$

$$M_z = M_z^{(o)} + M_z^{(1)} + M_z^{(2)} + \qquad (2\ 8)$$

$M^{(n)}$ and $M_z^{(n)}$ are small quantities of the nth order Assuming that the driving function $H_1(t)$ represents but a small perturbation to the Hamiltonian of the NMR system, i e , applied excitation amplitude is far less than the static magnetic field, and that $M^{(n)}$ and $M_z^{(n)}$ are zero when $H_1(t) = 0$ Substituting eq.

(2 7) in (2 5) and (2 8) in (2 6) yields a set of equations of various orders, and representative equations of one order are

$$\frac{dM^{(n)}}{dt} + \left(\frac{1}{T_2} + j\gamma h\right) M^{(n)} = j\gamma H_1(t)\, M_z^{(n-1)} \qquad (2\ 9)$$

$$\frac{dM_z^{(n)}}{dt} \simeq -\gamma H_1(t)\ \mathrm{Im}\ M^{(n-1)} \qquad (2\ 10)$$

Thus, $M^{(n)}$ may be calculated in terms of $M_z^{(n-1)}$, $M_z^{(n-1)}$ in terms of $M^{(n-2)}$ and so on In other words, knowing the lowest-order terms, all other terms may, in principle be calculated from these recurrence relationships Quantities such as $\gamma H_1(t)\, M_z^{(n-1)}$ are of order n as $H_1(t)$ is small Consider first the zeroth-order terms

$$\frac{dM^{(o)}}{dt} + \left(\frac{1}{T_2} + j\gamma h\right) M^{(o)} = 0 \qquad (2\ 11)$$

However, in the absence of any irradiation, we know that M must be zero and so $M^{(o)} = 0$ (No tilting of magnetization vector into transverse plane and hence no reading in that plane) The equation in M_z gives

$$\frac{dM_z^{(o)}}{dt} \simeq 0\ , \qquad (2\ 12)$$

and so $M_z^{(o)}$ is constant which may equate with the equilibrium magnetization $M_o(x,y,z)$ Moving on to the first-order equations

$$\frac{dM^{(1)}}{dt} + \left(\frac{1}{T_2} + j\gamma h\right) M^{(1)} = j\gamma H_1(t)\, M_z^{(o)} = j\gamma H_1(t)\, M_o(x,y,z) \qquad (2\ 13)$$

This equation will be analyzed in considerable detail later

Also,

$$\frac{dM_z^{(1)}}{dt} \simeq -\gamma H_1(t)\, M^{(o)} = 0$$

This is a repeat of eq (2 12), but as $M_z^{(1)}$ is zero for $H_1(t) = 0$, it must be zero all times

$$\left(\quad M_z = M_z^{(o)} + M_z^{(1)} + \cdots \right.$$

$$\left. = M_o + 0 + \quad \right)$$

The second-order equations thus give

$$\frac{dM^{(2)}}{dt} + \left(\frac{1}{T_2} + j\gamma h\right) M^{(2)} = 0 \qquad \left(\quad M_z^{(1)} = 0 \right)$$

and here $M^{(2)}$ is zero for the same reasons

$$\frac{dM_z^{(2)}}{dt} \simeq -\gamma H_1(t)\ \mathrm{Im}\ M^{(1)}$$

Hence

$$M_z^{(2)} \simeq \int_{-\infty}^{t} -\gamma H_1(t)\, M_y^{(1)}\, dt$$

This term describes the departure of the magnetization from the equilibrium alignment along the z-axis Finally, the third order equation in M is

$$\frac{dM^{(3)}}{dt} + \left(\frac{1}{T_2} + j\gamma h\right) M^{(3)} = j\gamma H_1(t)\, M_z^{(2)}$$

and this expression describes the deviation of M from the linear behaviour of Eq (2 13)

Equation (2 13) describes a linear differential equation, and so the technique of linear systems analysis may be brought to bear on the problem of solving the equation [19] Hence,

$$M^{(1)} = e^{-\int_o^t \left(\frac{1}{T_2} + j\gamma h\right)dt'} \int_o^t j\gamma H_1(t_1)\, M_o(x,y,z)\, e^{\int_o^{t_1}\left(\frac{1}{T_2}+j\gamma h\right)dt'} dt_1 \qquad (2\ 14)$$

writing Eq (2 14) in detail we have

$$M^{(1)} = \exp\left[-\int_o^t \frac{1}{T_2} + j\gamma H_o + j\gamma x G_x(t') + j\gamma y G_y(t') + j\gamma z G_z(t')\, dt'\right]$$
$$\int_o^t j\gamma H_1(t_1)\, M_o(x,y,z) \exp\left[\int_o^{t_1} \frac{1}{T_2} - j\gamma H_o + j\gamma x G_x(t')\right.$$
$$\left. + j\gamma y G_y(t') + j\gamma z G_z(t')\, dt'\right] dt_1 \qquad (2\ 15)$$

Substitution of $t_1 - t = \tau$ yields

$$M^{(1)} = -j\gamma M_o(x,y,z) \int_o^t H_1(t-\tau)\, e^{-\tau\left(\frac{1}{T_2} + j\gamma H_o\right)}$$
$$e^{j\gamma x \int_t^{t-\tau} G_x(t')dt'}\, e^{j\gamma y \int_t^{t-\tau} G_y(t')dt'}\, e^{j\gamma z \int_t^{t-\tau} G_z(t')dt'} \qquad (2\ 16)$$

With (2 16) as the basic solution for remaining terms can be written, at least in principle Hoult [12] showed in computer simulation studies that under the assumption that when input excitation is small enough to tilt magnetization vector from static field alignment by an angle less than 30^{o} the effect of $M^{(3)}$ and higher order terms can be neglected and the dominant term is only $M^{(1)}$ Our further analysis takes only $M^{(1)}$ term into consideration

Hence, receiver coil output is spatial integral of $M^{(1)}$ term only and estimation of spatial information(spin density) on this output is a fair approximation

Similar solution (Eq (2 16)) can be arrived at using Kinariwala's approach [10] This approach is to decompose the square matrix represented under Eq (2 1) into two matrices, $A_o(t)$ and $A_1(t)$, where $A_o(t)$ satisfies the commutative condition $A_1(t)$ is interpreted as a perturbation upon $A_o(t)$ This method is very similar to Hoult's perturbation analysis, except whole matrix is considered instead of individual terms

CHAPTER 3

SIGNAL PROCESSING

3 1 NMR DETECTION SYSTEM

Expression for $\bar{M}(t)$, the nuclear magnetization is derived in the previous chapter It is the function of the NMR detection system, the receiver, to detect $\bar{M}(t)$ and generate an output signal $y(t)$ The receiver coil, which usually surrounds the sample, is an antenna which picks up the fluctuating nuclear magnetization of the sample and converts it to a fluctuating output voltage $V(t)$, where

$$V(t) = - \frac{d}{dt} \int \bar{M}(t,\bar{x}) \quad B_c(\bar{x}) \, d\bar{x}$$

The function $B_c(\bar{x})$ describes the sensitivity of the receiver coil at different points in space The primary objective of receiver coil design is to prescribe wire placements so that $B_c(\bar{x})$ has the largest possible transverse component The longitudinal component of $B_c(\bar{x})$ contributes little to the output voltage, and can be ignored This is a result of the fact that the time derivative of $M_z(t,\bar{x})$ is much less than that of transverse component $M_z(t,\bar{x})$ decays exponentially with the time constant T_1 typically 0 1 to 1 S, while the transverse component is oscillatory with a period of typically, 0 05 to 0 2 μs Let

$$B_c(\bar{x}) = au_x + bu_y$$

where a and b are fixed, but unknown constants

Instead of V(t), time integral of V(t) (let this be y'(t)) is considered which leads to mathematical simplicity Hence

$$y'(t) = \int (aM_x(t,\bar{x}) + b\, M_y(t,\bar{x}))\, d\bar{x}$$

$$= \int \mathrm{Re}((a-jb)\, M(t,\bar{x}))\, d\bar{x}$$

Making the replacements $a = K\cos\phi'$ and $b = -K\sin\phi'$

$$y'(t) = -\int \mathrm{Re}\, Ke^{j\phi}\, M(t,\bar{x})\, d\bar{x}$$

$$= -\int \mathrm{Re}\, K\, M(t,\bar{x})\, d\bar{x}$$

where K is complex arbitrary constant

Dropping the constant part we have $\int M(t,\bar{x})\, d\bar{x}$, and let this be represented by y"(t) In detail y"(t) can be written from Eq (2 16) as

$$y''(t) = \iiint M_o(x,y,z) \int_o^t H_1(t-\tau)\, e^{-\tau(\frac{1}{T_2} + j\gamma H_o)}$$

$$e^{j\gamma x \int_t^{t-\tau} G_x(t')dt'}\; e^{j\gamma y \int_t^{t-\tau} G_y(t')dt'}$$

$$e^{j\gamma z \int_t^{t-\tau} G_z(t')dt'}\, d\tau\, dxdydz$$

By quadrature phase detection $j\gamma H_o$ term can be eliminated Let y" (t) after quadrature phase detection be represented as y(t) With the definitions

$$u(t,\tau) = \gamma \int_{t}^{t-\tau} G_x(t')\, dt'$$

$$v(t,\tau) = \gamma \int_{t}^{t-\tau} G_y(t')\, dt'$$

and

$$w(t,\tau) = \gamma \int_{t}^{t-\tau} G_z(t')\, dt'$$

$y(t)$ can be written as

$$y(t) = \int_{-\infty}^{+\infty}\int\int M_o(x,y,z) \int_{o}^{t} H_1(t-\tau)\, e^{-\tau/T_2}\, e^{jxu(t,\tau)}$$

$$e^{jyv(t,\tau)}\, e^{jzw(t,\tau)}\, d\tau\, dx\, dy\, dz \qquad (3\ 1)$$

The imaging experiment is treated in terms of system analysis The underlying concept is that of a black box which is tested with an input signal $H_1(t)$ and responds with an output signal $y(t)$ The feature to be revealed in this imaging experiment is the spin density distribution of the biological sample as a function of space coordinates For T_1 and T_2 imaging a different methodology has to be adopted From Eq (3 1) it can be seen that four variables (shape of the RF input signal, and the modulations of magnetic x,y,z gradients) are at our disposal Consequently the imaging experiment may be treated as a four-input time invariant system or a one-input, time dependent system, when the time dependence is imposed by the magnetic field gradients and the one input is the RF signal More specifically, the NMR imaging experiment is described in terms of a linear time-dependent system

Broadly imaging modalities may be classified as being 'reconstructive' or 'non-reconstructive' The former technique is an adaptation of the principles of image generation in X-ray CT, and involves reconstruction of the image in its entirety from integral projections

Modalities in which an image reconstruction scheme is not involved are classified as non-reconstructive These can be classified in two different ways

(i) Based on dimension 1-Dimension, 2-Dimension, 3-Dimension Using appropriate excitation signal and gradients, it is possible to excite desired section in spin density volume (Fig 4 5)

(ii) Based on pulse sequence Varieties of pulse sequences are in usage A specific method known as Kumar, Welti, and Ernst (KWE) direct Fourier NMR tomographic imaging is discussed below

The RF pulse and gradient sequence used in KWE method is illustrated in Fig. 3 1 and commences at t = 0 with a short nonselective RF pulse to nutate some or all of the equilibrium magnetization into the x,y plane This is then followed by periods t_x and t_y of variable length during which linear magnetic field gradients G_x,G_y are applied in x and y directions respectively Finally, the FID is sampled in the presence of a z-gradient G_z for a time $t_z = t-(t_x+t_y)$

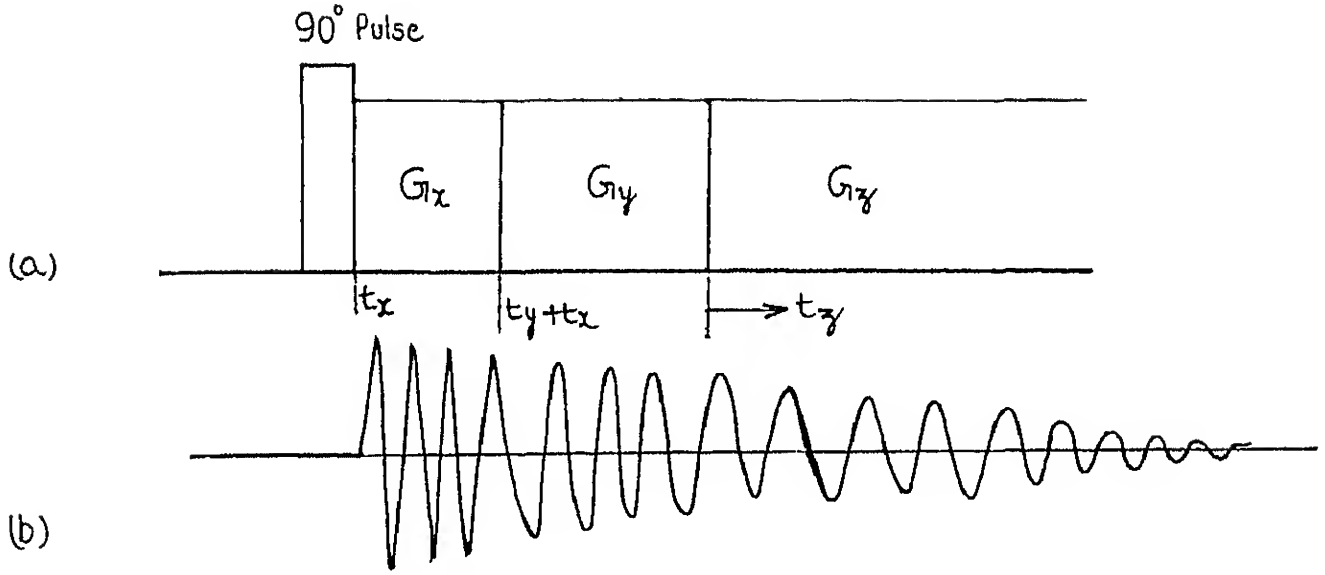


Fig. 3.1(a) Pulse sequence used in N.M.R. imaging method
(b) FID signal

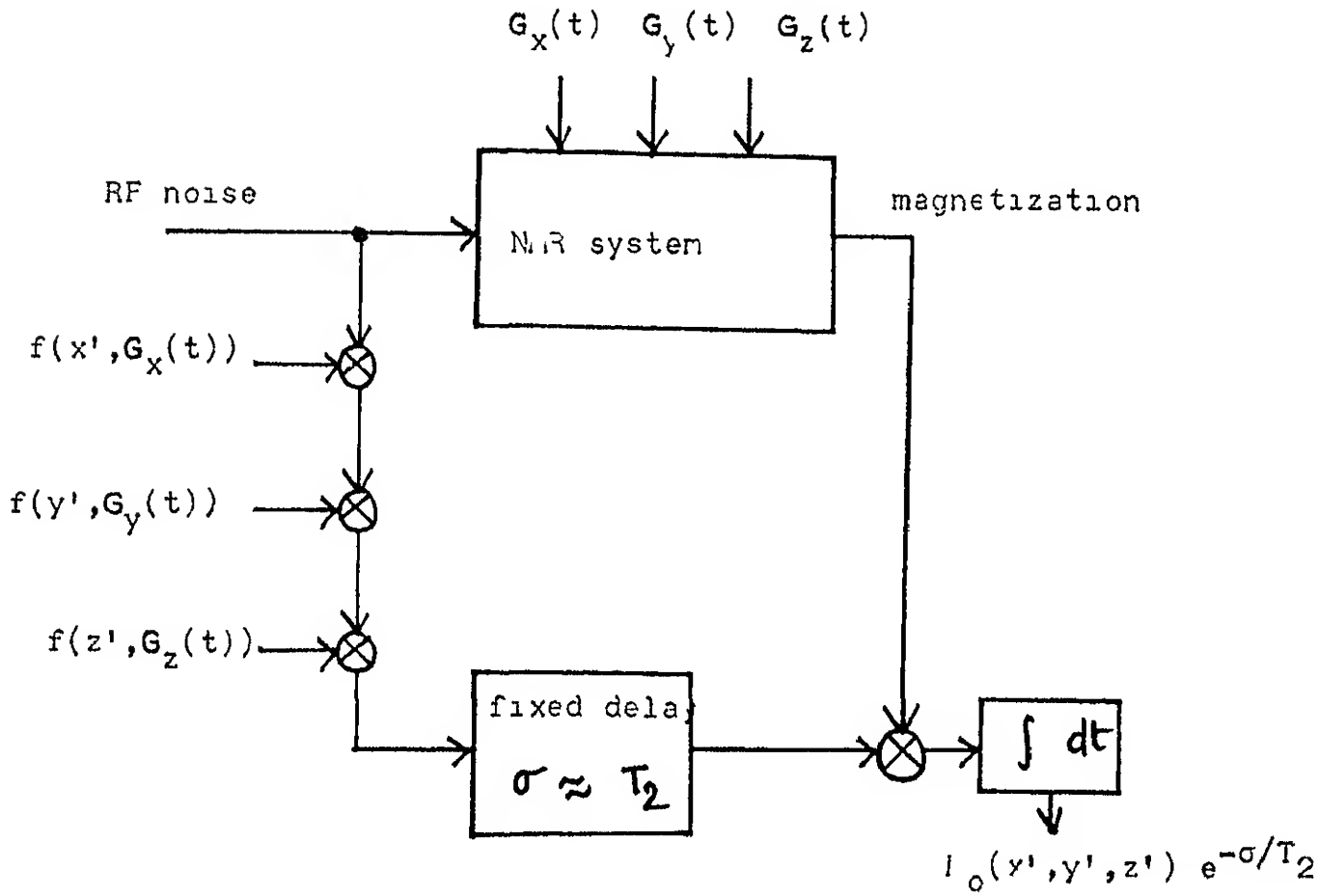


Fig. 3.2 Cross correlation scheme in NMR imaging

The z component of the magnetic field is therefore given by

$$\begin{aligned} &H_o + G_x x && 0 < t < t_x \\ &H_o + G_y y && t_x < t < t_x + t_y \\ &H_o + G_z z && t_x + t_y < t \end{aligned}$$

and the response of the spin system $y(t)$ will be a function of t_x and t_y, as well as t_z, i e , $y(\bar{t}) = y(t_x, t_y, t_z)$ The experiment is repeated for a full set of regularly stepped or incremented t_x and t_y values allowing a three-dimensional image of the object to be reconstructed as discussed below

The observed signal $y(\bar{t})$ can be written from Eq (3 1) as

$$y(\bar{t}) = \iiint M_o(x,y,z)\, e^{j\gamma G_x x t_x}\, e^{j\gamma G_y y t}\, e^{j\gamma G_z z t}\, e^{-(t_x+t_y+t_z)/T_2}\, dx\, dy\, dz$$

Neglecting the effect of T_2, $M_o(x,y,z)$ can be obtained by inverse 3-D Fourier transform as given below

$$M_o(x',y',z') = \iiint y(t_x,t_y,t_z)\, e^{-j\gamma G_x x' t_x}\, e^{-j\gamma G_y y' t_y}\, e^{-j\gamma G_z z' t_z}\, dt_x\, dt_y\, dt_z$$

3.2 STOCHASTIC NMR IMAGING [2]

Stochastic excitation denotes a noise-modulated RF input signal, which is supplied for the entire length of the imaging experiment RF noise is conveniently generated by a sequence of RF pulses with random flip angles The pulses are separated by

the sampling interval Δt For each input pulse one value of the response is sampled The stochastic RF excitation delivers its excitation energy to the NMR system within a characteristic relaxation time T_1 While pulsed excitation concentrates the same energy into a single pulse of length δt

The spin density is retrieved from cross correlation of input and output records The longer these records, the better will be achievable signal to noise ratio (SNR) of the spin density In stochastic system analysis the input signal usually is white noise with zero mean so that the autocorrelation of the real-valued input is proportional to a delta function

$$\lim_{T\to\infty} \frac{1}{T} \int_0^T H_1(t)\, H_1(t-\sigma)\, dt = \mu_2 \quad (\sigma) \qquad (3\ 2)$$

The proportionality constant μ_2 is the second moment of the power of the input

For time-invariant systems the linear impulse response function can be retrieved from cross-correlation of input and output signals according to

$$\lim_{T\to\infty} \frac{1}{T} \int_0^T y(t)\, H_1(t-\sigma)\, dt = \mu_2\, M'(\sigma) \qquad (3\ 3)$$

where $M'(t)$ is impulse response,

$$M'(t) = \int \bar{M}_o(\bar{x}) \exp\left[-\frac{t}{T_2} + j\gamma\bar{x} \int_o^t \bar{G}_x(t')\, dt'\right] d\bar{x} ,$$

with $M'(t < 0) = 0$ The cross-correlation is a time average of the product of the system response with the delayed input signal The result is the linear impulse response as a function of the delay σ In case of time-dependent systems this concept cannot be applied However, for the imaging system the goal is to retrieve its time-invariant part but not the complete time dependent impulse response Since the time dependent part is known, the system response $y(t)$ can be correlated with the product of all the time-dependent factors in the response integral

$$P(t-\sigma) = e^{-jx' u(t,\sigma)} \; e^{-jy' v(t,-\sigma)} \; e^{-jz' w(t,\sigma)} \; H_1(t-\sigma) \quad (3\,4)$$

The primed coordinates as well as σ become variables in the cross-correlation Its evaluation yields the spin density as a function of the cross-correlation variables. The complete scheme is shown in Fig 3 2 [3]

$$\lim_{T\to\infty} \frac{1}{T} \int_0^T y(t)\, P(t-\sigma)\, dt$$

$$= \iiint M_o(x,y,z) \int_0^\infty e^{-\tau/T_2} \lim_{T\to\infty} \frac{1}{T} \int_0^T e^{jxu(t,\tau)}$$

$$e^{-jx'u(t,\sigma)} \; e^{jyv(t,\tau)} \; e^{-jy'v(t,\sigma)} \; e^{-jz'w(t,\sigma)} \; e^{jzw(t,\tau)}$$

$$H_1(t-\tau)\, H_1(t-\sigma)\, dt\, d\tau\, dx\, dy\, dz \qquad (3.5)$$

$$= (2\pi)^3 \mu_2 \iiint M_o(x,y,z) \int_0^\infty e^{-\tau/T_2} \delta(x-x') \delta(y-y') \delta(z-z')$$

$$\delta(\tau-\sigma)\, d\tau\, dx\, dy\, dz \qquad (3\ 6)$$

$$= (2\pi)^3 \mu_2 M_o(x',y',z') \quad e^{-\sigma/T_2} \qquad (3.7)$$

In Eq (3 5) the cross-correlation has been written down explicitly but the order of integration has been changed The step from (3 5) to (3 6) involves the evaluation of the inner most integral The fourfold product of delta functions result from Eq (3 2), the definition of the delta function

$$\frac{1}{2\pi} \int_{-\infty}^{\infty} e^{jk(x-x_o)} \qquad = \delta(x-x_o) \qquad (3\ 8)$$

and a postulated orthogonality of the four functions in the product of Eq (3 4). The step from Eqs (3 6) to (3 7) is the evaluation of the delta functions

For the orthogonality to be satisfied three conditions must be met

Condition 1

The input signal $H_1(t)$ must have zero mean

$$\mu_1 = \lim_{T\to\infty} \frac{1}{T} \int_0^T H_1(t)\, dt = 0 \qquad (3\ 9)$$

Condition 2

The time integrals over the magnetic field gradients must always be different from zero

$$u(t,\tau),\ v(t,\tau),\ w(t,\tau) \neq 0 \qquad (3\ 10)$$

for all t and some given σ

Condition 3

The difference of the local phase angles in the different directions x,y,z should never vanish for all primed and unprimed values of x,y and z

$$\begin{aligned} xu(t,\sigma) - yv(t,\sigma) &\neq 0\ , \\ yv(t,\sigma) - zw(t,\sigma) &\neq 0\ , \\ zw(t,\sigma) - xu(t,\sigma) &\neq 0\ , \end{aligned} \qquad (3\ 11)$$

for all times t, the σ given, and all x,y,z

Condition 1 is trivial Condition 2 can be fulfilled by modulating a field gradient such that no gradient component ever changes sign The last condition is the most restricting one One way to observe it is by using significantly different field gradient in each direction x,y and z A different gradient scheme is simulated in next chapter for the case where the spin density shall be recovered at discrete coordinates x', y', and z'

The final result of the cross correlation (3 7) is the spin density as a function of space coordinates, scaled by the input power, μ_2, and a negative exponential involving the transverse relaxation time T_2 and the cross-correlation delay σ. Input power μ_2 cannot be made arbitrarily large in order to increase the system response, since eventually the system will become nonlinear and the response will be damped However, there exists an input power level which yields a response maximum and consequently optimum SNR in the acquisition of the response

In comparison with pulsed FT NMR, stochastic resonance [4]

1) has the same sensitivity ,
2) allows an independent optimization of sensitivity and resolution in contrast to pulse FT NMR ;
3) reduces the problems in the receiving electronics associated with the large dynamic range of impulse signals,
4) reduces the peak excitation power such that wider dynamic ranges may be covered ,
5) admits coherent averaging of the system response in connection with cyclic excitation ,
6) will exhibit different saturation features In particular, the nonlinear response will involve double resonance effects;
7) investigates the spin system in dynamic equilibrium with the excitation and not the unperturbed thermodynamic equilibrium state

Stochastic excitation in NMR imaging has three advantages over conventional methods with sparsely pulse PF input

1) The excitation power is reduced by several orders of magnitude
2) Since no deterministic pulse cycle exists, every additionally acquired data point increases the quality of the entire image
3) For slowly moving objects, reconstruction of the image can be restricted to those data, during the acquisition of which the object was in approximately the same state

3 3 CHIRP IMAGING

In NMR imaging the quest for high spatial resolution is of fundamental concern in clinical applications. High resolution necessiates the use of strong magnetic field gradients. Conventional imaging schemes traditionally employ fast switching of gradients. Since the speed of switching imposes an upper limit on the usable gradient strength, it is desirable to search for imaging schemes which permit slow switching of field gradients admitting stronger gradients for improved resolution. NMR imaging with sinusoidal field gradient modulation [5],[9] seems to be an attractive solution.

In general an infinite number of gradient modulation functions ($G_x(t,\tau)$, $G_y(t,\tau)$, $G_z(t,\tau)$) are conceivable, which obey the three conditions, Eqs (3 9), (3 10) and (3 11). We

propose a new class of imaging scheme, viz , Chirp processing because of its simplicity in generating gradient functions and subsequent processing structure Chirp processing has been developed to provide a solution for the conflicting requirements of simultaneous long-range and high -resolution performance in radar systems [14] Chirp processing is often interpreted as an alternative to Fourier transform in signal processing

Consider Eq (3 1) After the application of $\pi/2$ pulse, $y(t)$ can be written as

$$y(t) = \iiint M_o(x,y,z)\, e^{-t/T_2}\, e^{jxu(t)}\, e^{jyv(t)}\, e^{jzw(t)}\, dx\, dy\, dz \qquad (3\ 12)$$

T_2 is a spatial function and hence cannot be brought outside the integral, but let us neglect the effect of T_2

Let the gradients be linear functionsof time as given below

$$G_x(t) = K_1 t \qquad (3.13)$$

$$G_y(t) = K_2 t \qquad (3\ 14)$$

$$G_z(t) = K_3 t \qquad (3\ 15)$$

and

$$u(t) = \int_0^t K_1 t'\, dt' = \frac{K_1 t^2}{2} \qquad (3\ 16)$$

Similarly $$v(t) = \frac{K_2 t^2}{2} \qquad (3\ 17)$$

$$w(t) = \frac{K_3 t^2}{2} \qquad (3\ 18)$$

where K_1, K_2 and K_3 are constants The three conditions Eqs (3 9), (3 10) and (3 11) are satisfied by this particular choice as per the constraints given below

1) Input signal $H_1(t)$ must have zero mean

$$\int_0^{T_P} \sin \omega t \, dt = 0$$

2) The time integrals over the magnetic field gradients must always be different from zero

Our choice of field gradients, which are linear functions of time satisfy this property (Eq (3 13), (3 14) and (3 15))

3) The difference of the local phase angles in different directions x,y,z should never vanish for all primed and unprimed values of x,y and z From Eq (3 11)

$$x\ u(t) - y\ v(t) \neq 0$$

$$\Rightarrow x\ K_1 \neq y\ K_2$$

This is possible to maintain except for some set of values of x, K_1, y, K_2 All points in x-y plane can be imaged except those lying on straight line $y = xK_1/K_2$ This problem can be

overcome by repeating the experiment with different Chirp rates, i e , K_1' and K_2' instead of K_1 and K_2 such that $K_1'/K_2' \neq K_1/K_2$

The constraints under this condition can be given as

$$xK_1 \neq yK_2 \neq zK_3$$

and $x'K_1 \neq y'K_2 \neq z'K_3$

Multiplying y'(t) (Eq (3 12)) with $e^{-jx'K_1t^2/2}\ e^{-jy'K_2t^2/2}\ x\ e^{-jz'K_3t^2/2}$ and integrating with respect to time amounts to

$$\iiint M_o(x,y,z) \int e^{jK_1(x-x')t^2/2}\ e^{jK_2(y-y')t^2/2}\ e^{jK_3(z-z')t^2/2}\ dt\ dx\ dy\ dz$$

In the inner integral each term behaves approximately like a delta function

Consider $d(x-x') = \int_o^\infty e^{jK_1(x-x')t^2/2}\ dt$ This can be written as

$$\int e^{t^2/2\sigma^2}$$

where $\sigma = \frac{1}{\sqrt{K_1(x-x')}}\ \frac{1}{e^{j\pi/4}}$

From the knowledge of Gaussian function this integral can be written as

$$\sqrt{\frac{\pi}{2}}\ \frac{1}{\sqrt{K_1}}\ \frac{1}{\sqrt{x-x'}}$$, neglecting the phase part

The above function behaves approximately like a delta function and be represented as d(x-x') When x = x' the denominator goes to zero and hence the total quantity to ∞ When x' ≠ x then the function value is finite, ideally speaking it should be zero x' >> x of x' << x the function becomes zero But such cases are not encountered due to finite dimensions of the specimen

Since conditions 1,2 and 3 are satisfied the four integral function reduces to K" $M_o(x',y',z')$, where K" is a constant With different x',y',z' the whole specimen can be imaged Simulation work carried out in Chapter 4 justifies this method The processing can be made real time, at least in theory with an array of chirp multipliers and integrators

CHAPTER 4

SIMULATION

4 1 STOCHASTIC NMR IMAGING SIMULATION

The excitation $H_1(t)$ can be realized with either a white sequence of pulses or continuous noise with zero mean The particular distribution function is of no influence for evaluation of a linearly driven system In particular a binary sequence of flip angles will be sufficient Such sequences are easy to generate with feedback shift registers [15]

4 1 1 Discretization of the Spin Density

The second orthogonality condition is fulfilled by choosing distribution functions for the field gradients G_x, G_y, G_z with non zero mean

In consideration of the third orthogonality condition it will be assumed for now that the measured response derives from a spin density, which, contrary to reality, is defined only on a discrete set of equidistant coordinate points x,y,z This set of points shall be the same as the one on which the spin density $M_o(x',y',z')$ will be reconstructed by cross-correlation

To avoid overlap of values, an interleaving set of discrete coordinates is constructed by setting

$$x' = (n_x + \alpha_x)\ a\ ,$$

$$y' = (n_y + \alpha_y)\ a\ ,$$

$$z' = (n_z + \alpha_y)\ a\ ,$$

where n_i is an integer, $0 < \alpha_i < 1$ for $i = x,y,z$ and the dimension of a discrete spin density element or picture cell is specified by 'a' Similarly a discrete range of values is introduced for the gradient integrals

$$u = (n_u + \alpha_u)g$$

$$v = (n_v + \alpha_v)g$$

$$w = (n_w + \alpha_w)g$$

where n_i is an integer, $0 < \alpha_i < 1$ for $i = u,v,w$ and the product of the minimum field gradient step and the gradient switching interval Δt is denoted by g Without loss of generality the field gradient switching interval will be assumed to be identical to the sampling interval, and hence the same symbol The unit of g is rad/m The value of g measures the minimum precession phase per picture cell which is acquired during one gradient step of length Δt For a symmetrical distribution of gradient numbers n_i, the mean field gradients are $g_i = \alpha_i\ g/\Delta t$

Evaluation of one of equations (3 11) provides insight into a choice for $\alpha_x, \alpha_y, \alpha_z$ and $\alpha_u, \alpha_v, \alpha_w$

$$x'u - y'v = [(n_x+\alpha_x)(n_u+\alpha_u) - (n_y+\alpha_y)(n_v+\alpha_v)]\ a\ g$$

The orthogonality condition states, that the right-hand side must be different from zero for all values of n_x, n_y, n_z and n_u, n_v, n_w Setting $\alpha_x = \alpha_u$, $\alpha_y = \alpha_v$, $\alpha_z = \alpha_w$ it can be seen that

$$x'u - y'v = [n_x n_u + (n_x+n_u)\ \alpha_x + \alpha_x^2 - (n_y+n_v)\ \alpha_y - \alpha_y^2]\ a\ g$$

with

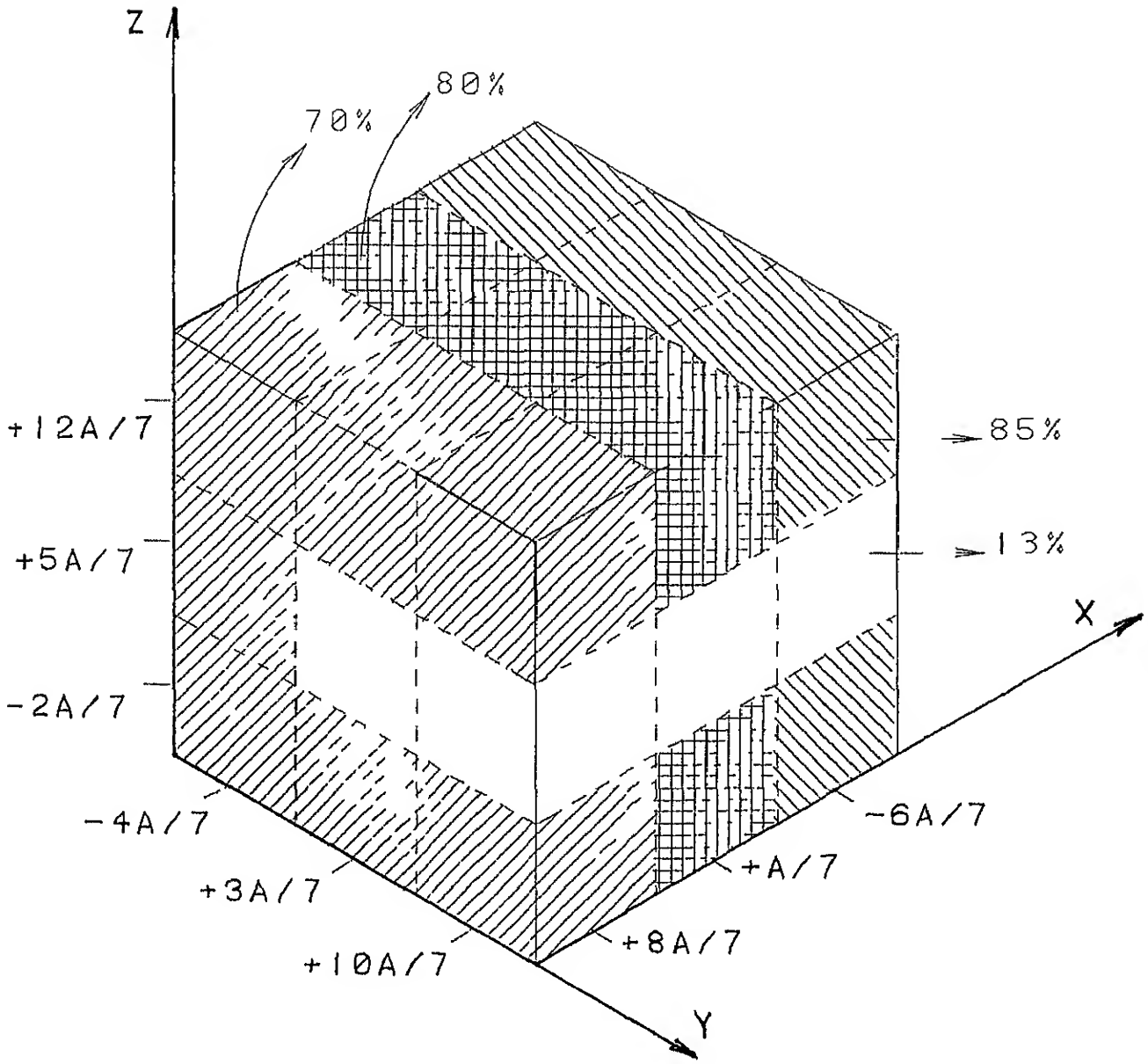
$$\begin{aligned} \alpha_x &= \alpha_u = 1/7, \\ \alpha_y &= \alpha_v = 3/7, \\ \alpha_z &= \alpha_w = 5/7 \end{aligned} \qquad (4\ 1)$$

the third orthogonality condition is fulfilled by the interleaving coordinate and gradient integral values according to Eqs (3 10), (3 11), and (4 1)

4 1 2 Numerical Simulation

The linear response has been calculated according to Eqs (3 1), (3 10), and (4 1) The spin density was defined on a 3x3x3 mesh (Fig 4 1) with values reflecting proton densities of living tissues The transverse relaxation time was T_2 = 0 1 sec (As mentioned earlier this should be a

Fig 41 NMR 3-D SPIN DENSITY



spatial function, but here it is assumed to be constant) To keep the computation times at an acceptable level, the product of spatial resolution and precession was set to ag = 2 5 rad The sampling interval was 0 01 sec For each sampling interval new field gradient values and a new excitation value were derived from a 20 stage binary feedback shift register The excitation could assume two values, $H_1 = [-1/2, +1/2]$ with equal probability, while the field gradient numbers could assume four values, $n_x, n_y, n_z = [-3/2, -1/2, +1/2, +3/2]$ with probabilities 1/8, 3/8, 3/8, and 1/8 The field gradient numbers were obtained by shifting the feedback register thrice and adding the binary outputs of each shift

The cross-correlation was executed for a 5x5x5 point mesh of the spin density $M_o(x',y',z')$ and a time shift $\sigma = T_2$ The result is shown in Fig 4 2 in terms of slices showing y'z' planes for different values of x' The numbers denote the retrieved spin density values percent The background noise results from the finite sample record length

For a random data ($0\ 0 \leq M_o(x,y,z) \leq 1\ 0$) also the simulation is carried out The assumed spin density and imaged values are shown in Fig 4 3 All the spin densities shown in this chapter are percentile

3	3	2	-4	-1
3	-1	9	-5	0
-12	7	4	-3	0
-6	-7	0	12	-1
-1	1	-7	2	3

$X' = -\frac{13}{7}\ a$

-1	5	-7	1	-2
0	-2	7	0	-2
5	0	6	2	0
-2	0	0	2	4
0	-2	7	0	-7

$X' = +\frac{15}{7}\ a$

7	-11	4	6	0
-3	62	71	80	5
4	84	12	10	-5
9	77	73	77	-8
4	-3	-7	3	6

$X' = +1/7a$

0	2	-6	-9	4
10	77	74	82	1
-9	79	13	11	7
2	86	94	88	-6
0	7	4	4	0

$X' = -\frac{6}{7}\ a$

-5	4	5	-5	0
8	59	76	70	-2
-17	73	12	12	-8
4	69	62	77	-5
6	-5	0	0	5

$X' = +\frac{8}{7}\ a$

Fig 4 2 Slices of retrieved spin density

$X = -\frac{6}{7}\,a$

19	32	6
73	10	71
60	18	63

$X = +\frac{1}{7}\,a$

4	37	40
67	53	79
39	88	43

$X = +\frac{8}{7}\,a$

1	39	36
13	29	13
67	18	68

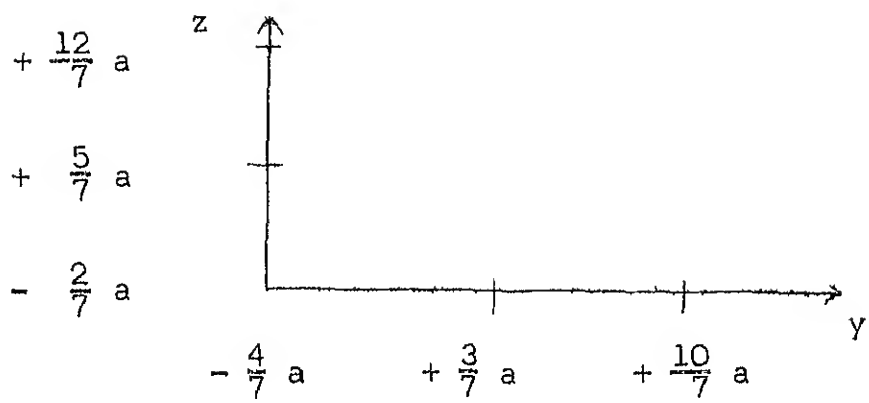
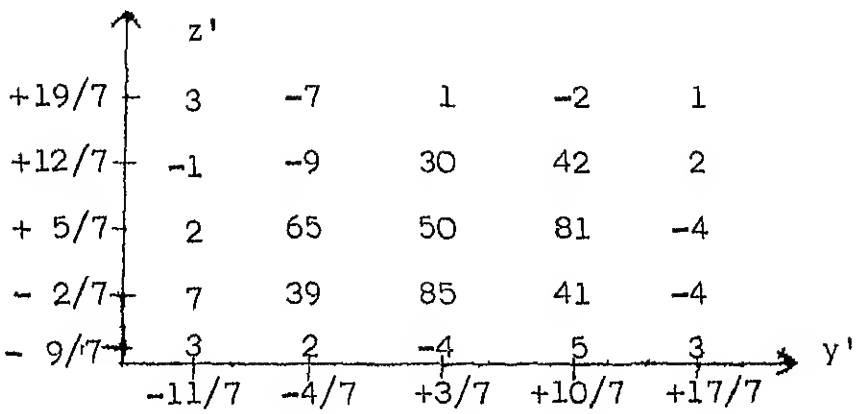


Fig 4 3(a) Spin density used in random simulation It is defined as a 3x3x3 array of cubic picture cells with dimension a

1	0	0	-4	-1	-7	3	-3	2	-1
3	-5	5	-3	0	0	1	9	- 5	-5
-4	5	2	-6	0	0	4	5	-2	-1
-5	-6	-1	10	0	-3	0	0	0	1
-3	2	-5	0	3	3	0	8	0	-6

$X' = -\frac{13}{7}\,a$ $\qquad$ $X' = +\frac{15}{7}\,a$



$X' = +\frac{1}{7}\,a$

1	0	-2	-2	0	-5	4	2	-6	-1
5	13	26	2	0	4	0	46	35	-1
-2	65	8	67	3	-7	16	26	12	-5
7	59	23	64	-3	1	67	7	72	-5
1	2	3	5	2	3	-5	0	0	2

$X' = -\frac{6}{7}\,a$ $\qquad$ $X' = +\frac{8}{7}\,a$

Fig. 4 3(b) Retrieved Spin Density Slices

4 2 CHIRP IMAGING SIMULATION

Under this imaging scheme, we assume that $\pi/2$ pulse is applied and then FID signal is recorded in the presence of linear time function gradients The pulse sequence is shown in Fig 4 4.

4 2 1 1-D Simulation

This process assumes that only a line of whole volume is excited and FID signal from other part is zero (Fig 4 5(b)) For one-dimension chirp imaging the FID signal can be written from Eq (3 12) as

$$y(t) = \int M_o(x)\, e^{jxK_1t^2/2}\, dx$$

Here T_2 effect is not taken into consideration When T_2 is assumed to be a constant its effect can be taken outside by multiplying $y(t)$ with e^{t/T_2} 65536 samples of $y(t)$ are multiplied with $e^{-jx'K_1t^2/2}$ and averaged over time to get $M_o(x')$ as shown below

$$\int y(t)\, e^{-jx'K_1t^2/2}\, dt$$

$$= \int M_o(x) \int e^{jK_1(x-x')\,t^2/2}\, dt\, dx$$

$$= \int M_o(x)\, d(x-x')\, dx$$

$$= M_o(x')$$

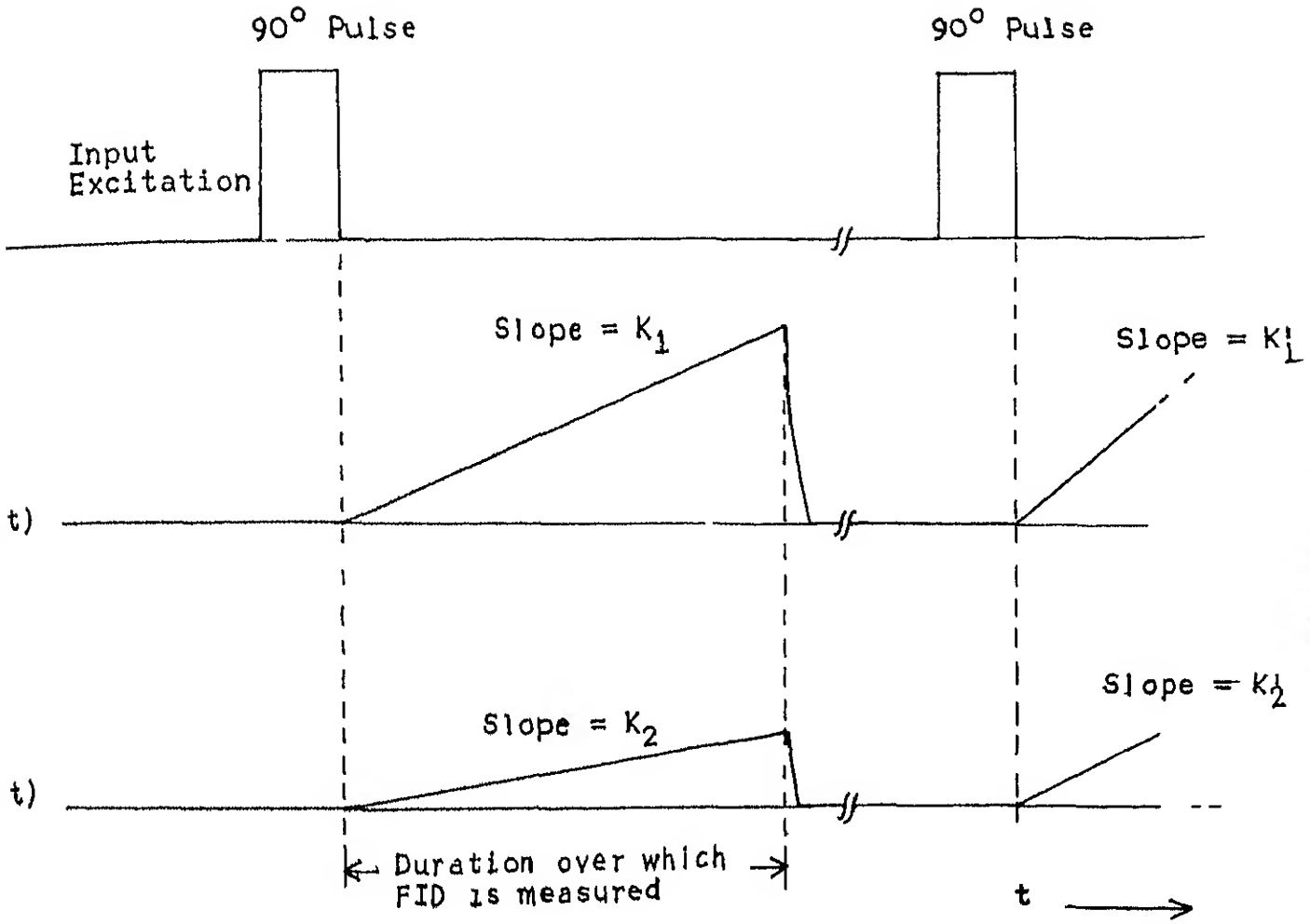


Fig 4.4 Pulse sequence for 2-dimensional chirp imaging

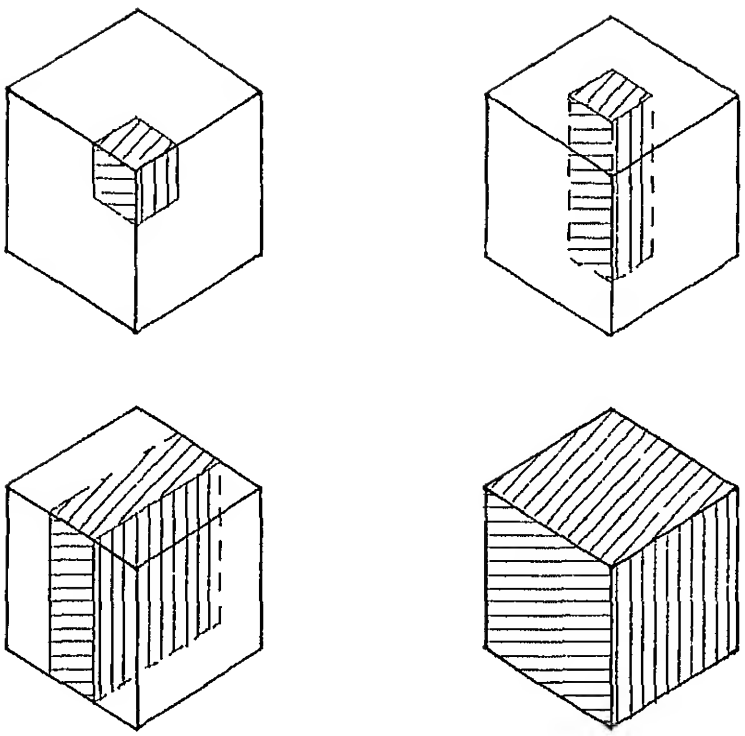


Fig 4.5

cheme of classification of imaging techniques (a) sequential
oint measurement (b) sequential line measurement, (c) sequential
lane measurement, and (d) simultaneous volume measurement

For simulation, $M_o(x)$ is taken as

$M_o(1) = 19/$, $M_o(2) = 73/$, $M_o(3) = 60/$, and imaged values are (using programs 5 and 6)

$M_o(1) = 25/$, $M_o(2) = 77/$, $M_o(3) = 64/$ and $M_o(1) = 5\ 6/$,

$M_o(4) = 6\ 6/$

Ideal values of $M_o(-1)$ and $M_o(4)$ should be zero These two values give idea of noise, where spin density is not present

4 2 2 2-D Simulation

This process assumes that only a slice of whole volume is excited [6], [20] and FID signal from remaining part is zero (Fig 4 5(c)) For two-dimension chirp imaging the FID signal can be written from Eq (3 12) as

$$y(t) = \iint M_o(x,y)\ e^{jK_1xt^2/2}\ e^{jK_2xt^2/2}\ dx\ dy$$

65536 samples of y(t) are multiplied with $e^{-jK_1x't^2/2}$ $e^{-jK_2y't^2/2}$ and averaged over time to get M(x',y') as given below

$$\int y(t)\ e^{-jK_1x't^2/2}\ e^{-jK_2y't^2/2}\ dt$$

$$= \iint M_o(x,y) \int e^{jK_1(x-x')t^2/2}\ e^{jK_2(y-y')t^2/2}\ dt\ dx\ dy$$

$$= \iint M_o(x,y)\ d(x-x')\ d(y-y')\ dx\ dy$$

$$= M_o(x',y')$$

$M_o(x,y)$ is taken as slices (Fig 4 6) of spin density shown in Fig 4 1 The imaged values (Programs 7 and 8) are shown in Fig 4 7

Programs 7 and 8 are ran for random 2-D (slice)spin density Results are given in Fig 4 8 The first line in this slice is the same data ran for 1-D chirp imaging (Section 4 2 1) The imaged values for the first line are different from 1-D chirp imaged values which clearly indicates the correlation effect of two impulses, viz , $\delta(x-x')$ and $\delta(y-y')$

4 3 DETAILS OF PROGRAMS

The simulation work is carried out on DEC-1090 computer system Storage of 65536 complex values of FID signal requires about 2800 blocks The following gives details of programs The listings are given in Appendix

Program 1 Simulates stochastic magnetization -
CPU time 6 hrs 5 mts 24 55 secs

Program 2 Cross-correlation for stochastic NMR tomography -
0 34 50 72

Program 3 Formated printing of input

Program 4 Formated printing of output

Program 5 Simulates 1-D chirp magnetization

Program 6 1-D chirp imaging

Program 7 Simulates 2-D magnetization - 0 2 53 12

Program 8 2-D chirp imaging -07 9 36

85	85	85	
85	13	13	Slice 1
85	85	85	
80	80	80	
80	13	13	Slice 2
80	80	80	
70	70	70	
70	13	13	Slice 3
70	13	13	

Fig 4 6 Slices of 3x3x3 spin density shown in Fig 4 1

4	14	12	12	4	
4	96	93	93	4	
4	98	27	26	4	Slice 1
4	97	93	93	4	
4	14	12	12	4	
4	13	12	11	4	
4	91	88	87	4	
4	92	26	25	4	Slice 2
4	91	88	87	4	
4	14	12	11	4	
3	12	10	10	3	
3	79	77	76	3	
3	80	24	24	3	Slice 3
3	79	77	76	3	
3	12	10	10	3	

Fig 4 7 Imaged slices

19	73	60
32	10	18
6	71	63

Fig 4 8(a) 3x3 Input slice (spin density) to Program 7

2	5	9	8	2
2	23	78	65	2
2	35	20	27	2
2	11	76	68	2
2	4	9	8	2

Fig 4 8(b) 5x5 Imaged slice through 2D chirp imaging

CHAPTER 5

CONCLUSIONS

NMR techniques can be applied for imaging the protonic tissue matter in the whole body The image can be obtained by a suitable set of gradient and RF signals and performing the appropriate signal processing The image can be obtained point by point or line by line or slice by slice or total volume Notable among the schemes are Fourier transform (FT) techniques and the correlatioh techniques We have used PN sequence as the RF signal and spatially linear gradients as suggested by Blumich The FID signal was simulated on the computer for a 3x3 matrix of 3 slices of spin density The simulation of the FID signal took about 6 hours CPU time on DEC-1090 computer, and the processing on FID signal to recover the image about 30 minutes Due to this large computer times involved larger matrix sizes, larger number of slices could not be tried out A new schemo of obtaining the image through the chirp processing is implemented In this scheme an RF signal is applied for a short duration followed by application of linear spatially and temporally varying gradient fields The FID signal starting from the point of termination of RF signal is simulated, and the associated chirp processing is carried out

The reconstructed images were found to be in good agreement with original spin density The chirp method is applied for 1-D and 2-D imaging satisfactorily But the results for 3-D were not encouraging

For obtaining the NMR image of 3x3x3 points from its FID signal approximately 1,769,472 complex multiplications and additions have to be performed by computer, which require a large computational time The time required under the modalities involving FT techniques can be significantly cut down using Fast Fourier transform algorithms However, chirp processing cannot be made faster due to lack of fast algorithms It appears possible in any case to reduce the storage requirement and CPU time by using series-parallel processing, and should be attempted

A detailed examination of the 3-D chirp is required to identify the reasons for unsatisfactory results we obtained We have not applied any windows on the data to study their effects on side lobe levels It appears possible to use appropriate window to improve the side lobe characteristics as done in conventional signal processing on finite data

REFERENCES

1 Bloch F , 'Nuclear Induction', Physics Review, vol 70, pp 460-474, 1946

2 Blumich Bernhard, 'Stochastic NMR Imaging', Journal of Magnetic Resonance 60, pp 37-45, 1984

3 Blumich Bernhard and Spiess H W , 'NMR Imaging with Sinusoidal Field Gradient Modulation', Diskussion-stagung der Fachgruppe Magnetische Pesonanz-spektroskopie der GDCh, 30 9-2 10 1985, Heidelberg

4 Blumich Beinhard, 'Stochastic NMR Spectroscopy', Bulletin of Magnetic Resonance, vol 7, No 1, pp 5-26

5 Blumich Bernhard and Spiess H W , 'NMR Imaging with Incommensurate Sampling and Gradient Modulation Rates', Journal of Magnetic Resonance, January 1986

6 Caprihan Arving, 'Effect of Amplitude Modulation on Selective Excitation in NMR Imaging', IEEE Transactions on Medical Imaging, vol MI-2, No 4, pp 169-175, December 1983

7 Cho Z H , Kim H S , Song H B , and James Cumming, 'Fourier Transform Nuclear Magnetic Resonance Tomographic Imaging', Proceedings of the IEEE, vol 70, No 10, pp 1152-1173, October 1982

8 Damadian Raymond, 'Tumor Detection by NMR', Science, vol 171, No 3976, pp 1151-1153, 19 March 1971

9 Feiner L F and Locher P P , 'On NMR Spin Imaging by Magnetic Field Modulation', Applied Physics 22, pp 257 271, 1980

10 Derusso P M , Roy R J , and Close C M , State Variables for Engineers, John Wiley and Sons, Inc , 1967

11 Hinshaw Waldo S , and Lent Arnol H , 'An Introduction to NMR Imaging From the Bloch Equation to the Imaging Equation', Proceedings of the IEEE, vol 71, No 3, pp 338-350, March 1983

12 Hoult D I , 'The Solution of the Bloch Equations in the Presence of a Varying B_1 Field - An Approach to Selective Pulse Analysis', Journal of Magnetic Resonance 35, pp 69-86, 1979

13 House Waylon V , 'Introduction to the Principles of NMR', IEEE Transactions on Nuclear Science, vol NS-27, No 3, pp 1220-1226, June 1980

14 Klauder J R , Price A C , Darlington S , and Albersheim W J , 'The Theory and Design of Chirp Radars', The Bell System Technical Journal, vol 39, No 4, pp 745-808, July 1960

15 Korn C.A , Random Process, Simulation and Measurement, McGraw-Hill, New York, 1966

16 Lauterbur P C , 'Image Formation by Induced Local Interactions Examples Employing Nuclear Magnetic Resonance', Nature, vol 242, pp 190-191, March 16,1973

17 Locher P R , 'Computer Simulation of Selective Excitation in NMR Imaging', Philosophical Transactions of Royal Society of London, B 289, pp 537-542, 1980

18 Mansfield P and Morris P G , 'NMR Imaging in Biomedicine', Supplement 2, in advances in Magnetic Resonance, Waugh J S , Ed New York Academic, 1982

19 Schwarz Ralph J and Friedland Bernard, Linear Systems, McGraw-Hill Book Company, 1965

20 Smith M R , 'RF Pulse Shapes for Selective Excitation in NMR Imaging', IEEE Transactions on Medical Imaging, vol MI-4, No 2, pp 79-83, June 1985

APPENDIX


```
************************
*       PROGRAM 1      *
************************
*       THIS PROGRAM SIMULATES STOCHASTIC MAGNETIZATION
*
*       SUMMARY:
*       The theory of this program is discussed in chapter4. NMR
*        system requires stochastic excitation (+1/2,-1/2) and random
*       field gradient values as given in chapter4.The assumed
*       3-Dimension Spin Density is given by RO(x,y,z). This can
*       have values from 0% to 100%. Another variable parameter
*       is the cell width (AG). This value is fixed at 2.5 keeping
*       computational complexity in view. The x,y,z coordinates
*       are decided by CX,CY,CZ and FX,FY,FZ. The sampled values of
*       FID are stored in RESP(1) to RESP(65536)(the continuous
*       FID is sampled at 0.01 sec. spacing).Dimension argument
*       is not provided to save the space. The values are sequentially
*       generated  from 1 to 65536.These values are directly used along
*       with the knowledge of input excitation values and gradient
*       function values.These values are also generated in a
*       sequence(1 to 65536) without using a dimension declaration
*       to save the space. This program requires about 7 hours
*       CPU time and occupies 2770 Blocks storage( ON DEC-1090
*       COMPUTER SYSTEM).
*..........................................................................
        COMMON/PHSE/ISROLL
        COMMON/PARM/RO(3,3,3),ITAU,S,R,IPRES,IEND
        COMMON/SHIFT/GRX(50),GRY(50),GRZ(50),IEX(50),IREG(21),EOR
        COMMON/CONST/CX,CY,CZ,FX,FY,FZ,AG
        INTEGER IREG,EOR,IEX,IRAN,GRX,GRY,GRZ
        COMPLEX S,R,RESP(64)
*       MAIN PROGRAM
        CALL INITIA
        READ(23,*),ISROLL,IPRES,(IREG(J),J=1,21),(GRX(J),J=1,50),
        1(GRY(J),J=1,50),(GRZ(J),J=1,50),(IEX(J),J=1,50)
        DO 500 I=901,1024
        DO 300 J=1,64
        CALL MAGN
300     RESP(J)=R
500     WRITE(24,*)(RESP(J),J=1,64)
        WRITE(26,*),ISROLL,IPRES,(IREG(J),J=1,21),(GRX(J),J=1,50),
        1(GRY(J),J=1,50),(GRZ(J),J=1,50),(IEX(J),J=1,50)
        STOP
        END
*
        SUBROUTINE INITIA
*
        COMMON/PHSE/ISROLL
        COMMON/PARM/RO(3,3,3),ITAU,S,R,IPRES,IEND
        COMMON/SHIFT/GRX(50),GRY(50),GRZ(50),IEX(50),IREG(21),EOR
        COMMON/CONST/CX,CY,CZ,FX,FY,FZ,AG
        INTEGER IREG,EOR,IEX,IRAN,GRX,GRY,GRZ
        COMPLEX S,R
*       CELL WIDTH IN RAD/2
        AG=2.5/2.
*       [illegible] SET CONSTANTS
        CX=1./7.
        CY=1./7.
        CZ=1./7.
        FZ=[illegible]
        FY=[illegible]
        FX=[illegible]
*       SHIFT REGISTER
        DO 10 I=1,21
10      IREG(I)=1
*       TIME KEEPING
        ISROLL=50
        IPRES=[illegible]
*       SPIN DENSITY CONTRAST:85%,80%,70%,13%
```

```
      DO 101 J=1,3
      DO 101 K=1,3
101   RO(1,J,K)=0.85
      DO 102 J=1,3
      DO 102 K=1,3
102   RO(2,J,K)=0.80
      DO 103 J=1,3
      DO 103 K=1,3
103   RO(3,J,K)=0.70
      DO 110 I=1,3
      DO 110 J=2,3
110   RO(I,J,2)=0.13
      END
*
      SUBROUTINE MAGN
*
      COMMON/PHSE/ISROLL
      COMMON/PARM/RO(3,3,3),ITAU,S,R,IPRES,IEND
      COMMON/SHIFT/GRX(50),GRY(50),GRZ(50),IEX(50),IREG(21),EOR
      COMMON/CONST/CX,CY,CZ,FX,FY,FZ,AG
      INTEGER IREG,EOR,IEX,IRAN,GRX,GRY,GRZ
      COMPLEX S,R
*     CONVOLUTION   INTEGRAL
      CALL PRESNT
      R=0.
      DO 400 ITAU=1,IEND
      IO=ISROLL+1-ITAU
      IF(IO.LE.0) IO=50+IO
*     SPACE INTEGRALS
      FTIME=(1.-ITAU)/10.
      S=0.
      GX=GRX(IO)+CX
      GY=GRY(IO)+CY
      GZ=GRZ(IO)+CZ
      DO 300 IX=1,3
      DO 300 IY=1,3
      DO 300 IZ=1,3
      S=S+RO(IX,IY,IZ)*CEXP(CMPLX(FTIME,0*AG))
300   CONTINUE
*     TIME INTEGRAL
400   R=R+S*IEX(IO)
      END
*
      SUBROUTINE PRESNT
*
      COMMON/PHSE/ISROLL
      COMMON/PARM/RO(3,3,3),ITAU,S,R,IPRES,IEND
      COMMON/SHIFT/GRX(50),GRY(50),GRZ(50),IEX(50),IREG(21),EOR
      COMMON/CONST/CX,CY,CZ,FX,FY,FZ,AG
*     IGX, IGY, IGZ ARE DOUBLE PRECISION
      INTEGER IREG,EOR,IEX,IRAN,GRX,GRY,GRZ,IGX,IGY,IGZ
      DOUBLE PRECISION
*     GRADIENTS
      IPRES=IPRES+1
      ISROLL=ISROLL+1
      IF(ISROLL.GT.50) ISROLL=1
      IGX=IRAN(1)
      IGY=IRAN(2)
      IGZ=IRAN(3)
      GRX(ISROLL)=IGX
      GRY(ISROLL)=IGY
      GRZ(ISROLL)=IGZ
*     FREQ PARAM
      IEX(ISROLL)=IRAN(4)
*     SQUARE GRADIENT INTEGRAL TABLE
      IEND=50
      IF(IPRES.GT.IEND) IEND=IPRES
      IF(IEND=1
      IF(J.GE.4)GO TO 400
```

```
      DO 300  ITAU=1,J
      IO=ISROLL-ITAU
      IF(IO.LE.0) IO=50+IO
      GRX(IO)=GRX(IO)+IGX
      GRY(IO)=GRY(IO)+IGY
300   GRZ(IO)=GRZ(IO)+IGZ
400   END
*
      FUNCTION IRAN(K)
*
      COMMON/PHSE/ISROLL
      COMMON/PARM/RO(3,3,3),ITAU,S,R,IPRES,IEND
      COMMON/SHIFT/GRX(50),GRY(50),GRZ(50),IEX(50),IREG(21),EOR
      COMMON/CONST/CX,CY,CZ,FX,FY,FZ,AG
      INTEGER IREG,EOR,IEX,IRAN,GRX,GRY,GRZ
      COMPLEX S,R
*
      IRAN=0
      DO 200 J=1,K
      DO 100 I=1,20
100   IREG(22-I)=IREG(21-I)
      EOR=IREG(21)+IREG(18)
      IREG(1)=EOR-2*(EOR/2)
200   IRAN=IRAN+IREG(20)
      IRAN=2.*(IRAN-K/2.)
      END
```

```
*************************
*       PROGRAM 2       *
*************************
C THIS PROGRAM PERFORMS CROSS-CORRELATION FOR STOCHASTIC NMR TOMOGRAPHY
*
*       SUMMARY:
*       This program reads the sampled values of FID(RESP(1) to
*       RESP(65536)) generated by program 1.This program generates
*       same input excitation and field gradient values to
*       reconstruct the Spin Density.These values are generated in
*       a sequence from a RAN (random) subprogram to avoid
*       dimension declaration in order to save space.The theory
*       of this program is discussed in chapter4. The Spin Density
*       coordinates are decided by CX,CY,CZ and FX,FY,FZ. The
*       reconstructed Spin Density is a 3-Dimension function.
*       The input to this program is the output of program 1.
*       This program requires about 30 minutes CPU time. The
*       output is a 5X5X5 matrix. Comparision with the 3X3X3 input
*       gives the idea of deviation error(comparing same points)
*       and truncation error and noise(comparing the points
*       for which Spin Density values are not assigned).Same
*       cell width value (AG=2.5) is to be used.The assumed
*       delay(SIGMA) is 0.1 sec.
*.....
      COMMON/PASE/ISROLL
      COMMON/PARA/RO(5,5,5),RESP(64),IPRES,IEND
      COMMON/SHIFT/GRX(50),GRY(50),GRZ(50),IEX(50),IREG(21),EOR
      COMMON/CONST/CX,CY,CZ,FX,FY,FZ,AG,ISIGMA
      INTEGER IREG,EOR,IEX,IRAN,GRX,GRY,GRZ
      COMPLEX RESP
      MAIN PROGRAM
      CALL INITIA
      DO 500 I=1,1023
      READ(21,*)(RESP(J),J=1,64)
      CALL CROSS
500   F=(EXP((ISIGMA-1.)/10.))/(64.*1024.)
      DO 500 I=1,5
      DO 500 J=1,5
      DO 500 K=1,5
550   RO(I,J,K)=RO(I,J,K)*F
600   WRITE(22,*)(RO(I,J,K),K=1,5)
      STOP
      END
*
*       SUBROUTINE INITIA
*
      SUBROUTINE INITIA
      COMMON/PASE/ISROLL
      COMMON/PARA/RO(5,5,5),RESP(64),IPRES,IEND
      COMMON/SHIFT/GRX(50),GRY(50),GRZ(50),IEX(50),IREG(21),EOR
      COMMON/CONST/CX,CY,CZ,FX,FY,FZ,AG,ISIGMA
      INTEGER IREG,EOR,IEX,IRAN,GRX,GRY,GRZ
      COMPLEX RESP
*     SEED VALUE FOR RAN/2
      IRAN=2.5E2
      AG=2.5E2
      DELAY= SIGMA T2=10
      ISIGMA=0
*     OFFSET CONSTANTS
      CX=1./11.
      CY=1./11.
      CZ=5./11.
      FX=1./11.
      FY=1./11.
      FZ=1./11.
*     SHIFT REGISTER
      DO 10 I=1,21
10    IREG(I)=1
*     LOOK FOR SPIKING
      IOREG=652
      IPRES=0
```

```
*       Solid Density
        DO 2, I=1,5
        DO 20 J=1,5
        DO 20 K=1,5
20      RO(I,J,K)=0.
*       START-UP PHASE
        READ(21,*)(RESP(J),J=1,64)
        DO 100 J=1,64
100     CALL PRESNT
        END
*
        SUBROUTINE CROSS
*
        COMMON/PHSE/ISROLL
        COMMON/PARM/RO(5,5,5),RESP(64),IPRES,IEND
        COMMON/SHIFT/GRX(50),GRY(50),GRZ(50),IEX(50),IREG(21),EOR
        COMMON/CONST/CX,CY,CZ,FX,FY,FZ,AG,ISIGMA
        INTEGER IREG,EOR,IEX,IRAN,GRX,GRY,GRZ
*       NOTE: AM IS COMPLEX
        COMPLEX RESP,AM
*       CROSS CORRELATION
        DO 300 I=1,64
        CALL PRESNT
        IO=ISROLL+1-ISIGMA
        IF(IO.GE.0) IO=50+IO
        AM=RESP(I)*IEX(IO)
        GX=GRX(IO)+CX
        GY=GRY(IO)+CY
        GZ=GRZ(IO)+CZ
        DO 300 IX=1,5
        DO 300 IY=1,5
        DO 300 IZ=1,5
        D=(IX-FX)*GZ+(IY-FY)*GY+(IZ-FZ)*GZ
300     RO(IX,IY,IZ)=RO(IX,IY,IZ)+AM*CEXP(CMPLX(0.,-D*AG))
        END
*
        SUBROUTINE PRESNT
*
        COMMON/PHSE/ISROLL
        COMMON/PARM/RO(5,5,5),RESP(64),IPRES,IEND
        COMMON/SHIFT/GRX(50),GRY(50),GRZ(50),IEX(50),IREG(21),EOR
        COMMON/CONST/CX,CY,CZ,FX,FY,FZ,AG,ISIGMA
*       NOTE: FX,FY,FZ ARE DOUBLE PRECISION
        INTEGER IREG,EOR,IEX,IRAN,GRX,GRY,GRZ
        COMPLEX RESP
*       GRADIENTS
        IPRES=IPRES+1
        ISROLL=ISROLL+1
        IF(ISROLL.GT.50) ISROLL=1
        IGX=IRAN(3)
        IGY=IRAN(3)
        IGZ=IRAN(3)
        GRX(ISROLL)=IGX
        GRY(ISROLL)=IGY
        GRZ(ISROLL)=IGZ
*       EXCITATION
        IEX(ISROLL)=IRAN(1)
*       UPDATE GRADIENT INTEGRAL VALUE
        IEND=ISIGMA
        IF(IPRES.GT.1540) IEND=IPRES
        J=IEND-1
        IF(J.LE.0) GO TO 400
        DO 300 IRAN=1,J
        IO=ISROLL-IRAN
        IF(IO.LE.0) IO=50+IO
        GRX(IO)=GRX(IO)+IGX
        GRY(IO)=GRY(IO)+IGY
300     GRZ(IO)=GRZ(IO)+IGZ
```

```
400   END
*
      FUNCTION IRAN(K)
*
      COMMON/PHSE/ISROLL
      COMMON/PARM/RO(5,5,5),RESP(64),IPRES,IEND
      COMMON/SHIFT/GRX(50),GRY(50),GRZ(50),IEX(50),IREG(21),EOR
      COMMON/CONST/CX,CY,CZ,FX,FY,FZ,AG,ISIGMA
      INTEGER IREG,EOR,IEX,IRAN,GRX,GRY,GRZ
      COMPLEX RESP
*
      IRAN=0
      DO 200 J=1,K
      DO 100 I=1,20
100   IREG(22-I)=IREG(21-I)
      EOR=IREG(21)+IREG(18)
      IREG(1)=EOR-2*(EOR/2)
200   IRAN=IRAN+IREG(20)
      IRAN=2.*(IRAN-K/2.)
      END
```

```
**************************
*       PROGRAM 3        *
**************************
*THIS PROGRAM PRINTS THE RANDOM 3-D INPUT SPIN DENSITY IN  3(3X3) MATRICES
*.........................................................................
      DIMENSION RO(5,5,5)
      DO 101 I=1,3
      DO 101 J=1,3
      DO 101 K=1,3
      R12=RAN(31)
      R22=R12*100
      IIII=R22
101   RO(I,J,K)=IIII
      DO 102 I=1,3
      WRITE(20,99)
99    FORMAT(///)
      DO 102 J=1,3
102   WRITE(20,100)(RO(I,J,K),K=1,3)
100   FORMAT(' ',3('   ',F4.1)/)
      STOP
      END
```

```
**************************
*       PROGRAM 5        *
**************************
*       THIS PROGRAM GENERATES 65536 SAMPLE POINTS
*       (FID SIGNAL) OF 1-D CHIRP IMAGING
*       The theory of this program is discussed in chapter4 in 1-D
*       Chirp Imaging. The Spin Density is considered to be 1 Dimension.
*       The sampled values of FID are given sequentially from 1 to
*       65536.
*.....................................................................
      COMPLEX S
      DIMENSION RO(3)
      OPEN (UNIT=21,FILE='FOR21.DAT',DEVICE='DSKD')
*
*       FOLLOWING INITIALIZES SPIN-DENSITY AND CX,FX,AG etc.
*       AG: CELL WIDTH IN RAD/2
      AG=2.5/2.
*       DEF. SEI CONSTANTS
      CX=1.77
      FX=2.-CX
*       G: CHIRP RATE DENOTED AS K/2 IN THE ANALYSIS
      G=10
*       THRU' RAN FUNCTION 1-D RANDOM SPIN DENSITY IIS GENERATED HERE.
      DO 101 IS=1,3
      R12=RAN(1)
      R22=R12*100
      IIII=R22
      RO(IS)=IIII/100.0
101   CONTINUE
*
*       FID AT VARIOUS POINTS SAMPLED
*
      DO 200 II=1,65536
      T=II*G/65536.
*     FTIME=T/0.1
      GX=T*T
      S=0.
      DO 300 IX=1,3
      D=(IX-FX)*GX
      S=S+RO(IX)*CEXP(CMPLX(0.,D*AG))
300   CONTINUE
      WRITE(21,*)S
200   CONTINUE
      STOP
      END
```

```
**********************
*       PROGRAM 6      *
**********************
*       THIS PROGRAM PERFORMS INVERSE CHIRP PROCESSING (1-D IMAGING)*
*       FOR PROGRAM 5(1-D CHIRP PROCESSING)
*       The input to this program is the output of program 5.
*.....................................................................
        DIMENSION RO(5)
        COMPLEX FUN
*       CELL WIDTH IN RAD/2
        AG=2.5/2.
*       OFFSET CONSTANTS
        CX=1./7.
        FX=3.*CX
*       G:  CHIRP RATE, DENOTED AS K/2 IN ANALYSIS.
        G=10.
        DO 20 I=1,5
20      RO(I)=0.
*
        DO 40 II=1,65536
        READ(21,*)FUN
        I=II*G/65536.
        GX=I*T
        DO 30 IX=1,5
        D=(IX-FX)*GX
30      RO(IX)=RO(IX)+FUN*CEXP(CMPLX(0.,-D*AG))
40      CONTINUE
        E=1./(64.*1024.)
        DO 50 I=1,5
        RO(I)=RO(I)*E
50      WRITE(22,*)RO(I)
        STOP
        END
```

```
*************************
*       PROGRAM 7       *
*************************
*       THIS PROGRAM GENERATES 65536 SAPLE VALUES
*       (OF FID SIGNAL) OF 2-D CHIRP PROCESSING.
*       FOLLOWING INITIALIZES SPIN-DENSITY AND CX,FX,AG etc.
*.....................................................................
      COMPLEX S,RESP
      DIMENSION RO(5,5),RESP(64),R(29)
      OPEN (UNIT=21,FILE='FOR21.DAT',DEVICE='DSKD')
*
*     CELL WIDTH IN RAD/2
      AG=2.5/2.
*     OFFSET CONSTANTS
      CX=1./7.
      CY=3./7.
      FX=2.-CX
      FY=2.-CY
*     G1,G2 ARE CHIRP RATES DENOTED AS HALF OF K1,K2 RESPECTIVELY IN ANALYSIS
      G1=9.123456789
      G2=57.098765430
*
      IT=0
      K=0
      DO 10 IS=1,5
      DO 10 JS=1,5
      K=K+1
*     THIS PROGRAM GENERATES SPIN DENSITY SLICE VALUES RANDOMLY.
*     FOR EXECUTION THIS PROGRAM HAS TO BE LINKED WITH IMSL.
      CALL GGUBS(192,25,R)
      R12=R(K)
      R22=R12*100
      III=R22
      RO(IS,JS)=III/100.0
10    CONTINUE
*
*     FID AT VARIOUS POINTS SAMPLED
*
      DO 30 I=1,1024
      DO 30 J=1,64
      IT=IT+1
      GX=(IT*G1/65536.)**2
      GY=(IT*G2/65536.)**2
      S=0.
      DO 20 IX=1,5
      DO 20 IY=1,5
      X=(IX-FX)*GX+(IY-FY)*GY
20    S=S+RO(IX,IY)*CEXP(CMPLX(0.,0*AG))
      RESP(J)=S
30    WRITE(21,*)(RESP(J),J=1,64)
      STOP
      END
```

```
*************************
*       PROGRAM 8       *
*************************
*       THIS PROGRAM PERFORMS INVERSE 2-D CHIRP PROCESSING (2-D IMAGING)
*       FOR PROGRAM 7(2-D CHIRP PROCESSING)
*.....................................................................
      DIMENSION RO(7,7),FUN(64)
      COMPLEX FUN
*     CELL WIDTH IN RAD/2
      AG=2.5/2.
*     OFFSET CONSTANTS
      CX=1./7.
      CY=3./7.
      EX=3.*CX
      FY=3.*CY
*     G1,G2: CHIRP RATES DENOTED AS HALF OF K1,K2 IN ANALYSIS.
      G1=9.123456789
      G2=57.098765430
      II=0
*
      DO 10 I=1,7
      DO 10 J=1,7
10    RO(I,J)=0.
*
      DO 40 I=1,1024
      READ(21,*)(FUN(J),J=1,64)
      DO 30 J=1,64
      II=II+1
      GX=(II*G1/65536.)**2
      GY=(II*G2/65536.)**2
      DO 20 IX=1,7
      DO 20 IY=1,7
      D=(IX-CX)*GX+(IY-FY)*GY
20    RO(IX,IY)=RO(IX,IY)+FUN(J)*CEXP(CMPLX(0.,-D*AG))
30    CONTINUE
40    CONTINUE
      F=1./(64.*1024.)
      DO 50 I=1,7
      DO 50 J=1,7
      RO(I,J)=RO(I,J)*F
50    WRITE(22,*)RO(I,J)
      STOP
      END
```

EE-1986-M-KUM-SYS